



OPTIMUM PERFORMANCE OF IRREVERSIBLE STIRLING ENGINE WITH IMPERFECT REGENERATION

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Abstract—An optimal performance analysis is performed for a Stirling engine with heat transfer and imperfect regeneration irreversibilities. The relationship between the net power output and thermal efficiency of the engine is derived. Detailed numerical examples are given. The results obtained in this paper provide guidance to performance evaluation and design improvement for Stirling engines. Published by Elsevier Science Ltd.

Stirling cycle Imperfect regeneration Heat transfer Optimal efficiency

INTRODUCTION

Stirling engines are noteworthy for high efficiency, low noise, minimum vibration, durability, simple construction, ability to self-start etc. Being an external combustion engine, it utilizes a wide variety of energy resources: solar, atomic, geothermal, industrial waste heat and marsh-gas. Therefore, the engine has been used widely in energy resource utilization and power engineering [1–3].

The development of finite-time thermodynamics [4–8], a new discipline in modern thermodynamics, provides a powerful tool for performance analysis of practical engineering cycles. Several authors have studied the finite-time thermodynamic performance of the Stirling engine [7–13]. Many investigators [1, 7–13] have studied the effect of heat transfer on the power output of a Stirling engine. Petrescu [7, 9] obtained an optimal efficiency for maximum power output of a solar Stirling engine with imperfect regeneration. Extending these researches, a relationship between the net power output and efficiency for the Stirling engine with heat transfer and imperfect regeneration irreversibilities is derived in the following sections.

STIRLING ENGINE

The following assumptions are made for the Stirling engine model in this paper:

1. The cycle is made from two isothermal branches connected by two isochoric regenerating branches. The working fluid is an ideal gas. The engine operates between a constant-temperature heat source with temperature T_h and a heat sink with temperature T_c . The heats transferred into and out of the engine are:

$$Q_{in} = \alpha(T_h - T_1)t_1 \quad (1)$$

and

$$Q_{out} = \alpha(T_2 - T_c)t_2 \quad (2)$$

where α is the heat conductance between the engine and the reservoirs, T_1 and T_2 are the average temperatures of the working fluid in the expansion cavity and compression cavity, and t_1 and t_2 are the times associated with the external heat transfer processes, respectively.

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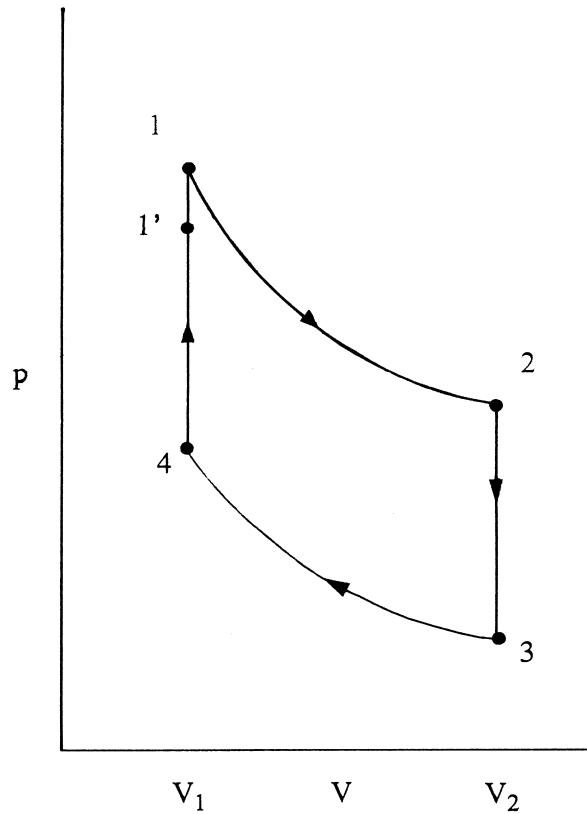


Fig. 1. Stirling engine cycle pressure-volume diagram.

2. There exists a finite-speed effect in the regenerating processes [10]. The regenerating processes proceed according to constant rates of temperature change, i.e.,

$$dT/dt = \pm(1/k_i), \quad i = 1, 2 \quad (3)$$

where the sign “+” and “−” correspond to temperature increase and decrease processes, and k_1 and k_2 are the positive, constant rates of the two regenerating processes. Note that $k_1 = k_2$ for an endoreversible cycle, and $k_1 \neq k_2$ for an internal irreversible cycle. For a non-uniform temperature change, $1/k_i$ is the average rate of temperature change.

3. Assume the heats regenerated in the two regenerating processes by taking into account the imperfect regeneration are unequal. The outlet state of the working fluid in the absorbing heat process inside the regenerator is at 1' and not at 1 as illustrated in Fig. 1. The imperfect regeneration coefficient is denoted by x , and the regeneration loss (ΔQ_R) is [7]

$$\Delta Q_R = xQ_{23} \quad (4)$$

where Q_{23} is the heat transferred out of the working fluid in the regeneration process 1–2. Hence, the heat transferred into the working fluid in the regeneration process 4–1' ($Q_{41'}$) is:

$$Q_{41} = (1 - x)Q_{23}. \quad (5)$$

OPTIMAL RELATION BETWEEN POWER OUTPUT AND EFFICIENCY

Based on the performance of the working fluid for the two endoreversible processes 1–2 and 3–4, we have

$$Q_{12} = nRT_1 \ln \lambda \quad (6)$$

$$Q_{34} = nRT_2 \ln \lambda \quad (7)$$

and

$$Q_{23} = nC_v(T_1 - T_2) \quad (8)$$

where n , R and C_v are the mole number, gas constant, and specific molar heat capacity of the working fluid, $\lambda = V_2/V_1$ is the compression ratio, and Q_{12} and Q_{34} are the heats transferred into and out of the engine for the endoreversible expansion and compression processes, respectively.

The heat absorbed by the working fluid in the expansion cavity for an internal irreversible cycle is more than that for an endoreversible cycle. The heat loss ΔQ_R , which flows from the regenerator into the compression cavity, is the internal heat leak. Therefore, the actual heats transferred from the heat source and to the heat sink for the irreversible engine are:

$$Q_{in} = Q_{12} + \Delta Q_R \quad (9)$$

and

$$Q_{out} = Q_{34} + \Delta Q_R \quad (10)$$

where Q_{in} and Q_{out} are the heats transferred into and out of the engine for the irreversible expansion and compression processes, respectively.

Combining equations (1), (2), (4) and (6)–(10) gives

$$t_1 = [nC_v(k-1)T_1 \ln \lambda + \alpha nC_v(T_1 - T_2)]/[\alpha(T_h - T_1)] \quad (11)$$

and

$$t_2 = [nC_v(k-1)T_2 \ln \lambda + \alpha nC_v(T_1 - T_2)]/[\alpha(T_2 - T_c)] \quad (12)$$

where k is the ratio of the specific heats.

Integrating equation (3) gives the regenerating times t_3 and t_4 as follows:

$$t_3 = k_1(T_1 - T_2) \quad (13)$$

and

$$t_4 = k_2(T_1 - T_2). \quad (14)$$

The cycle period (τ) is the sum of the individual process times

$$\tau = t_1 + t_2 + t_3 + t_4. \quad (15)$$

Conservation of energy requires

$$W = Q_{in} - Q_{out} = nR(T_1 - T_2) \ln \lambda. \quad (16)$$

The net power output (P) can be found by dividing the net work output (W) by the cycle period (τ)

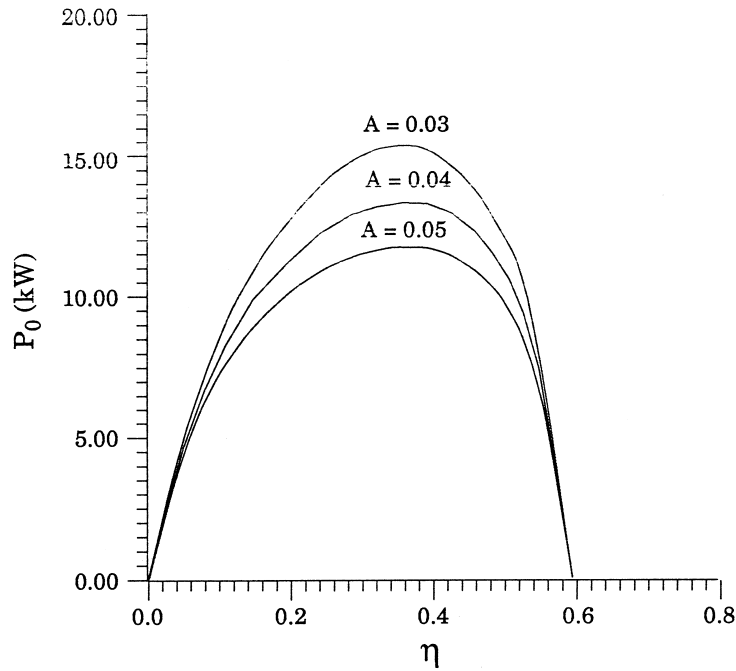
$$P = W/\tau = [nR(T_1 - T_2) \ln \lambda]/\tau. \quad (17)$$

Substituting equations (11)–(14) into equation (17) yields

$$P = \alpha(1-y)/\{[1 + \mu(1-y)]/(T_h - T_1) + [y + \mu(1-y)]/(yT_1 - T_c) + A(1-y)\} \quad (18)$$

where $y = T_1/T_2$, $A = \alpha(k_1 + k_2)/[nC_v(k-1)\ln \lambda]$, and $\mu = \alpha/[(k-1)\ln \lambda]$. The thermal efficiency (η) of the engine is

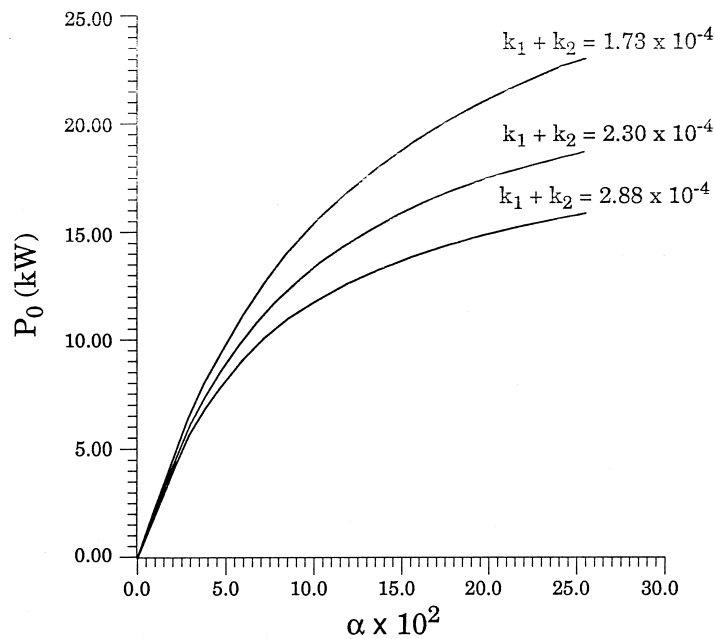
$$\eta = W/Q_{in} = (1-y)/[1 + \mu(1-y)]. \quad (19)$$

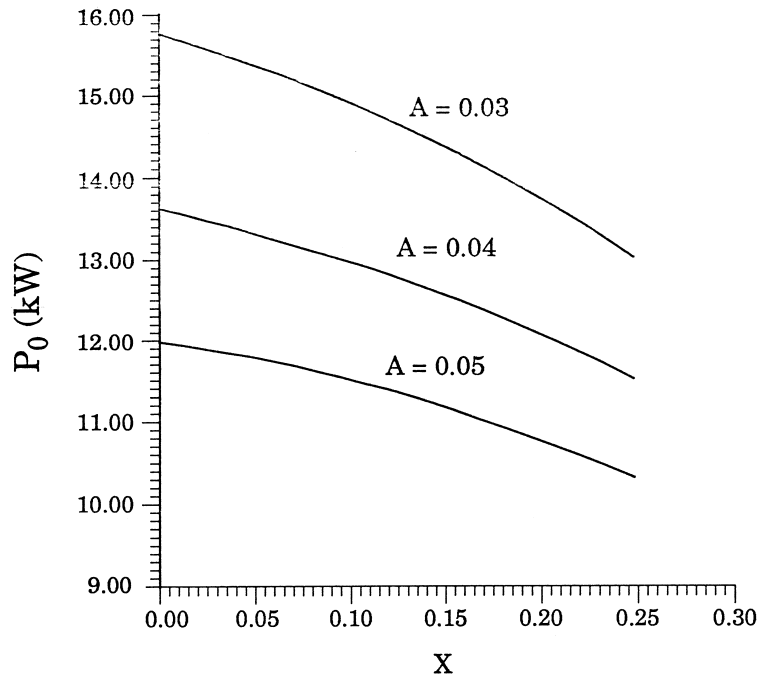
Fig. 2. Power output P_o versus efficiency η characteristic.

Substituting equation (19) into equation (18) gives

$$P = \alpha\eta / \{ [1/(T_h - T_1) + [(1 - \eta)(1 - \mu)] / [(1 - \mu\eta - \eta)T_1 - (1 - \mu\eta)T_c] + A\eta] \}. \quad (20)$$

Equation (20) gives the general relationship between power output and thermal efficiency of the Stirling engine. Taking the derivative of P with respect to T_1 and setting it equal to zero $[(\partial P / \partial T_1)_\eta = 0]$ at a fixed efficiency gives the optimal temperature of the warm working fluid

Fig. 3. Power output P_o versus heat conductance α .

Fig. 4. Power output P_o versus imperfect regeneration coefficient x .

$[(T_1)_o]$ and the optimal power output (P_o) in the following expressions:

$$(\partial P / \partial T_1)_\eta = 0 \quad (21)$$

$$(T_1)_o = [(b - b\eta)^{1/2} T_h + b T_c] / [1 + (b - b\eta)^{1/2}] \quad (22)$$

and

$$P_o = \alpha \eta \{ A \eta + [1 + (b - b\eta)^{1/2}]^2 / [T_h - b T_c] \}^{-1} \quad (23)$$

where $b = (1 - \mu\eta) / (1 - \eta - \eta)$.

Equation (23) is the major result of this analysis. It determines the optimum relationship between the net power output and the thermal efficiency. Equation (23) shows that the power output is zero when $\eta = 0$ or $\eta = (T_h - T_c) / [T_h + \mu(T_h - T_c)]$. Therefore, there exists a maximum power output (P_m). The maximum power output and the corresponding efficiency at the maximum power output (η_m) may be obtained from numerical calculation.

NUMERICAL EXAMPLE

The optimum power output versus efficiency characteristics and the maximum power output depend on α , $(k_1 + k_2)$, x , T_h and T_c . The P_o versus η curves with $T_h = 800$ K, $T_c = 300$ K, $\alpha = 1$ kW/K, $x = 0.05$, $n = 1$ mol, $k = 1.66$, $\lambda = 2$ and $A = 0.03, 0.04$ and 0.05 are plotted in Fig. 2. The efficiency bound η_m at the maximum power output point obtained from Fig. 2 is 0.36. η_m depends on x , α but is independent of $(k_1 + k_2)$.

The effects of heat conductance on the optimal power output P_o with $\eta = 0.36$, $(k_1 + k_2) = 0.000173, 0.000230$ and 0.000288 are shown in Fig. 3. It is shown that the optimal power output is a strong function of α and increases with the increases of α .

The effects of the imperfect regeneration on the optimal power output P_o with $\eta = 0.36$ and $A = 0.03, 0.04$ and 0.05 are plotted in Fig. 4. It is shown that the maximum power output decreases with the increase of imperfect regeneration coefficient x .

The results obtained above show that α , x and $(k_1 + k_2)$ (or A) are the important factors affecting the performance of the Stirling engine. The performance of the Stirling cycle due to internal irreversibility is very different from the performance of the endoreversible Stirling cycle.

CONCLUSION

The effects of heat transfer, regeneration time, and imperfect regeneration on the performance of the irreversible Stirling engine cycle are considered in this paper. The power output versus the efficiency characteristic, the optimal power output versus heat conductance characteristic, and the optimal power output versus imperfect regeneration coefficient characteristic are obtained. The analysis provides a new theoretical basis for the performance evaluation and improvement of practical Stirling engines.

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